Hawking radiation from four-dimensional Schwarzschild black holes in M theory

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Recently a method has been developed for relating four dimensional Schwarzschild black holes in M theory to near-extremal black holes in string theory with four charges, using suitably defined ''boosts'' and T dualities. We show that this method can be extended to obtain the emission rate of low energy massless scalars for the four dimensional Schwarzschild hole from the microscopic picture of radiation from the near extremal hole. [S0556-2821(99)04702-5]

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In the past couple of years there has been considerable progress in understanding the statistical basis of the thermodynamics of near-extremal five and four dimensional black holes in string theory [1]. Further, a simple effective model gives the correct low energy Hawking radiation from such holes. A natural question to ask is: Can we get the emission from neutral (i.e. Schwarzschild) holes as well?

Recently there has been some progress in understanding Schwarzschild black holes in M theory. The basic idea is to use 11-dimensional Lorentz invariance properties to relate Schwarzschild holes boosted along x^{11} to string theory states carrying Ramond-Ramond charges [2]. In [3] a concrete map was found which relates Schwarzschild strings and black p-branes. It was found that the precise relationship is not through a genuine boost in the compact direction, but through a boost in the covering space. The boosted coordinate is then re-compactified on a circle of radius which is related to the original radius by Lorentz contraction. This is not an exact symmetry of the theory, but provides a concrete map at the classical level. T dualities can generate other charges from the momentum charge, and a combination of boosts and T dualities may be used to relate Schwarzschild holes with other known black holes in string theory carrying charges, as in [4]. Specific maps which relate a five-dimensional Schwarzschild black hole with the standard five dimensional black hole in string theory with three large charges were given in [3]. Using similar steps one can map the four dimensional Schwarzschild hole to three sets of near-extremal 5-branes of M theory, intersecting in a common line along x^{11}, and carrying momentum along this direction [5]. This is a description of a four dimensional black hole with four charges in M theory [6]. The entropy of the latter near extremal hole is known to follow from a microscopic calculation [7,8], and thus we get the entropy of the neutral hole [5].

In the microscopic picture we can also get the emission from the above near extremal model [9], so one wonders if the maps allow us to predict the emission from neutral holes. We show in this paper that, using a general relation between absorption cross sections derived in [3], such is indeed the case, at leading order in the energy if we assume during the calculation that the radius of x^{11} is large enough. The important issue turns out to be the way the maps act on the emitted quantum while they transform the black hole—it turns out that the low energy scalars emitted from the neutral hole indeed map to quanta whose emission we can compute from the microscopics of the near extremal hole.

We will consider 11-dimensional M theory compactified on a space $T^p \times S^1$. The torus has sides $L_i, i=1,\ldots,p$ and the circle, which will be taken to be along the x^{11} direction, has a radius R. In this space a ''Schwarzschild string'' is a product of a Schwarzschild black hole in the noncompact $(10-p)$ dimensions and flat space along the $(p+1)$ compact dimensions. [This is thus a $(p+1)$ black brane. We call this a string since it extends along $x^{11}\equiv z$.] The 11-dimensional metric is

$$ds_{11}^2 = -\left(1-\left(\frac{r_0}{r}\right)^n\right)dt^2 + \frac{dr^2}{\left(1-\left(\frac{r_0}{r}\right)^n\right)} + r^2 d\Omega_{n+1} + dz^2 + \sum_{i=1}^{p} (dx^i)^2, \quad (1)$$

where $n=7-p$ and $r^2=\Sigma_{i=p+1}^9 (x^i)^2$. When the Schwarzschild radius is smaller than the radius R, a second object—the periodic Schwarzschild black hole—which is a periodic version of a $(11-p)$-dimensional Schwarzschild hole with x^{11} as one of the transverse directions—is entropically favorable. This is the subject of [2,10–12]. A proposal for the microscopic description from this phase has been given in terms of a gas of D0 branes [11,12].

The map which relates the Schwarzschild string to a charged hole consists of a boost in the covering space in which x^{11} is noncompact, but the other p directions are still compact

$$z' = z \cosh\alpha + t \sinh\alpha, t' = t \cosh\alpha + z \sinh\alpha. \quad (2)$$

The boosted coordinate z' has to be then compactified on a radius R' which is related to R by a standard Lorentz contraction

$$R' = R/\cosh\,\alpha. \tag{3}$$

By standard Kaluza-Klein reduction along z' (as in [13]) the resulting metric then represents a RR charged black hole in $(10-p)$ dimensions. Applying T duality along the p compact directions results in a black p-brane. The relationship (3) ensures that the energy and momenta transform correctly and the semiclassical entropy is kept invariant under the boosts [3]. For further application of these ideas to seven dimensional Schwarzschild holes see [14].

Furthermore it was shown in [3] that not only the entropy but the absorption cross sections (and hence Hawking radiation rates) of neutral and charged black holes may be related using these ''boosts'' at least when R and R' are large enough to allow us to ignore the quantization of momentum in this direction. If $\sigma(\omega,q,\vec{k};A)$ denotes the absorption cross section of some particle of energy ω, momentum q along x^{11}, and transverse momentum $\vec{k}$ by the black hole $\mathcal{A}$, then the absorption cross-section $\sigma'(\omega',q',\vec{k}';\mathcal{A}')$ of the transformed particle with energy-momentum given by $(\omega',q',\vec{k})$ by the transformed black hole $\mathcal{A}'$ is given by

$$\sigma'(\omega',q',\vec{k}';\mathcal{A}') = \frac{\omega}{\omega'}\,\sigma(\omega,q,\vec{k};\mathcal{A}),$$

$$q' = q\,\cosh\,\alpha + \omega\,\sinh\,\alpha, \quad \omega' = q\,\sinh\,\alpha + \omega\,\cosh\,\alpha. \tag{4}$$

The emission rate $\Gamma(k)$ is related to σ by

$$\Gamma(\omega,q,\vec{k}) = \frac{\sigma(\omega,q,\vec{k})}{e^{\xi}\mp 1}\,\frac{d^d\vec{k}}{(2\pi)^d}; \quad \xi = (\omega - q\phi)/T, \tag{5}$$

where T is the temperature of the black hole, and ϕ is the potential of the Kaluza-Klein gauge field at the horizon (in 10D language) and d is the number of transverse dimensions. The relation (4) does not depend on *how* the cross section is calculated, but follows from the fact that decay rates decrease by a time dilation factor. Hence if one had a microscopic derivation of the cross section for the charged black hole one may use this to obtain a microscopic derivation of the cross section for the neutral black hole. Relations similar to above have been used to predict the *total* emission rates in the D0 gas model in [12].

We now apply the above formula for four dimensional black holes. Consider a four dimensional Schwarzschild black hole with area $A_0 = 4\pi r_0^2$ and temperature $T_0 = 1/4\pi r_0$ tensored with T^7 [the metric is given by Eq. (1) with $p=6$] emitting some massless scalar, e.g. a longitudinal component of the metric h_{12} with some momentum k_0 along a transverse direction, say x^7. The energy of this particle is thus $\omega_0 = |k_0|$. The universal low energy absorption cross section is given by the area of the two dimensional horizon [15]

$$\sigma_0 = A_0, \tag{6}$$

while the parameter appearing in the thermal factor is $\xi_0 = \omega_0/T_0$.

Now perform the following operations.

(1) Boost along x^{11} by parameter α_1
(2) T dualize along (1234)
(3) Boost along x^{11} by parameter α_2
(4) T dualize along (1256)
(5) Boost along x^{11} by parameter α_3
(6) T dualize along (1234)
(7) Boost along x^{11} by parameter α_4
(8) T dualize along (1256)

The entropy is the same at all stages. At every step we will denote the string coupling by g_n, the string length by l_n, the radii of the torus by $L_i^{(n)}$ and the x^{11} radius by R_n where $n=1\cdots 8$. The first seven steps were used in [5]. For us, however, the last step is important.

With every boost by a parameter α the two dimensional horizon area and the temperature (in Planck units) changes as

$$A \to A' = A\,\cosh\,\alpha, \quad T \to T' = T/\cosh\,\alpha. \tag{7}$$

The absorption cross sections are related by Eq. (4). A T duality keeps the cross-section invariant, but changes the nature of the black hole as well as that of the emitted particle. Our strategy will be to first obtain a prediction for the semiclassical absorption cross section at the final stage, starting from the known semiclassical answer for the absorption cross section at the initial stage. Finally the former will be compared with a microscopic calculation performed at the last stage.

After the first step one has $R_1 = R/\cosh\,\alpha_1$. The black hole has x^{11} momentum which is a 0-brane charge in ten dimensional language. Its area A_1 and temperature T_1 and rank-1 potential at the horizon ϕ_1 are [3]

$$A_1 = A_0\cosh\,\alpha_1, \quad T_1 = T_0/\cosh\,\alpha_1, \quad \phi_1 = \tanh\,\alpha_1. \tag{8}$$

The emitted particle has energy ω_1 and momentum q_1 along x^{11}

$$\omega_1 = \omega_0\cosh\,\alpha_1, \quad q_1 = \omega_0\sinh\,\alpha_1, \tag{9}$$

while the transverse momentum is unchanged (in Planck units). In string theory this is a zero brane with quantized charge $Q_0 = q_1 R_1$. Note that the expression appearing in the thermal distribution function $\xi_0 = \omega_0/T_0 = (\omega_1 - q_1\phi_1)/T_1 = \xi_1$ appears in the correct form. Using Eqs. (4) and (8) the absorption cross section for the Hawking particle is

$$\sigma_1 = \sigma_0\frac{\omega_0}{\omega_1} = A_1\frac{\omega_1 - q_1\phi_1}{\omega_1}. \tag{10}$$

After the second transformation, the black hole is a collection of nonextremal D4 branes of string theory along

(1234), or longitudinal 5-branes of M theory along (1234, 11). Using standard T-duality formulas the radius of x^{11} in this M theory becomes

$$R_2 = R_1 \frac{l_1^4}{L_1^{(1)} L_2^{(1)} L_3^{(1)} L_4^{(1)}}. \tag{11}$$

The area and the temperature do not change, $A_2 = A_1, T_2 = T_1$.

The emitted particle is now an *extremal* 4-brane of string theory with some transverse motion, energy $\omega_2 = \omega_1$ and the quantized 4-brane charge is $Q_4 = Q_0$. To avoid explicit mention of higher form gauge fields, we will use an ''equivalent x^{11} momentum'' for this 4-brane, defined as follows. We perform T dualities which convert this 4-brane into a 0-brane. In this case these are T dualities along (1234). This results in a new underlying M theory with a x^{11} radius $\tilde{R}_2$ which may be easily calculated using standard T duality formulas to yield $\tilde{R}_2 = R_1$. The emitted particle then has a x^{11} momentum q_2:

$$q_2 \equiv Q_4 / \tilde{R}_2 = q_1. \tag{12}$$

We will use this ''equivalent x^{11} momentum'' in all the following steps and denote it by q_n and denote the corresponding M-theory radius by $\tilde{R}_n$. Written in terms of the new quantities the thermal factor is exactly what is expected, $\xi_2 = \xi_1 = (\omega_2 - q_2 \phi_1)/T_2$. The absorption cross-section is, of course unchanged, $\sigma_2 = \sigma_1$.

After the third step, the black hole is a D4 brane along (1234) with some 0-brane charge, or a nonextremal 5-brane with longitunal momentum in the language of M theory. Its area and temperature are

$$A_3 = A_2 \cosh\,\alpha_2, \quad T_3 = T_2/\cosh\,\alpha_2. \tag{13}$$

The crucial point is that the nature of the emitted particle does not change appreciably at this step. If the emitted four-brane was at rest this would have been an extremal five brane in M theory which is invariant under boosts in the longitudinal direction. This means that the metric produced by this object and its integer valued charge remains the same. Since the radius of x^{11} is Lorentz contracted, total energy is decreased by the same factor

$$\omega_3 = \omega_2/\cosh\,\alpha_2. \tag{14}$$

Once again we need to find the equivalent x^{11} momentum. One finds that $\tilde{R}_3 = \tilde{R}_2 \cosh\,\alpha_2$ so that

$$q_3 = q_2/\cosh\,\alpha_2. \tag{15}$$

Essentially the same conclusion holds when there is a small transverse momentum, as will be justified later. At this stage one has $\xi_3 = \xi_2 = (\omega_3 - q_3 \phi_1)/T_3$. Finally the cross section is

$$\sigma_3 = \sigma_2 \frac{\omega_2}{\omega_3} = A_3 \frac{\omega_3 - q_3 \phi_1}{\omega_3}, \tag{16}$$

where we have used Eqs. (14) and (15).

The remaining steps are repetitions of the above. For the even-numbered T-duality steps, $\omega_{2n} = \omega_{2n-1}$, $q_{2n} = q_{2n-1}$ and $\sigma_{2n} = \sigma_{2n-1}$ while for the odd number steps involving boosts $\omega_{2n+1} = \omega_{2n}/\cosh\,\alpha$, $q_{2n+1} = q_n/\cosh\,\alpha$, where α is the relevant boost parameter, while σ_{2n-1} is related to σ_{2n-2} by the relation (4). Note that from step (4) onwards the T dualities required to define the equivalent x^{11} momentum for the emitted particle are *not* the same as the T dualities in the previous step. Nevertheless, it is easy to check that the above relations continue to hold. The main point is that for low transverse momentum, the emitted particle carries only the charge which is imparted to it by the first boost α_1.

At the end of these steps we have a four dimensional black hole (in the noncompact space $x^7 \cdots x^9$) made of $D0$ branes along with three sets of intersecting D4 branes along (1234), (1256) and (3456) which is emitting D0 branes. The black hole is near-extremal when the boost parameters α_n are large. In the language of M theory we have three sets of five-branes intersecting along x^{11} and carrying some momentum along x^{11}, emitting particles which carry momentum along x^{11} equal to q_8 as well as a small transverse momentum. The black hole has a four dimensional area

$$A_8 = A_0 \cosh\,\alpha_1 \cosh\,\alpha_2 \cosh\,\alpha_3 \cosh\,\alpha_4, \tag{17}$$

and the energy and charge of the emitted particle are

$$\omega_8 = \frac{\omega_0 \cosh\,\alpha_1}{\Pi_{i=2}^4 \cosh\,\alpha_i}, \quad q_8 = \frac{\omega_0 \sinh\,\alpha_1}{\Pi_{i=2}^4 \cosh\,\alpha_i}. \tag{18}$$

The cross section obtained by the above procedure is

$$\sigma_8 = \sigma_0 \frac{\omega_0 \omega_2 \omega_4 \omega_6}{\omega_1 \omega_3 \omega_5 \omega_7} = A_8 \frac{\omega_8 - q_8 \phi_1}{\omega_8}. \tag{19}$$

Note that after all these steps, ϕ_1 has again become the potential due to the rank-1 gauge field at the horizon to which the emitted 0-brane couples. Equation (19) is precisely the semiclassical answer for emission from the four dimensional black hole with four charges [9].

In [9] an effective microscopic model for the above emission process has been proposed in the near extremal limit: this is a superconformal field theory with $c = 6$ on a line parallel to the intersection of the branes, but with a length equal to the charges times the length of this direction. This model correctly reproduces the semiclassical entropy. Furthermore, the emission rates for low energy scalars (in the noncompact four dimensional sense) which may carry some momentum along x^{11} was calculated in this microscopic theory along the lines of [16] and the result is exactly the semiclassical answer given by Eq. (19) (with $\phi_1 = \tanh\,\alpha_1 \sim 1$, since the hole is near-extremal).

We have related the low energy absorption cross section of uncharged minimal scalars in the neutral hole to the low energy charged scalar absorption for the four charge near extremal hole. Thus the microscopic calculation of [9] also provides a microscopic calculation of Hawking radiation from the four dimensional Schwarzschild hole.

Note that the agreement in absorption properties is not related to the fact that the cross section of neutral quanta for *both* neutral and charged holes is the area of the horizon and thus does not follow from the agreement of the entropies. Rather, in our calculation, we observe first that under the boost the neutral hole gets a charge, while at the same time the cross section becomes *smaller* than the area. This smaller cross section is however just the right one to represent the emission of the *charged* quanta from the charged hole. This pattern repeats under the T dualities or boosts, giving at the end a black hole carrying four charges, and emitting particles carrying one of the four charges (the momentum charge). The crucial point is that the various maps which relate the neutral to the near extremal hole also map, at the same time, the emitted particle to something which can be treated easily in the microscopic model.

It is important to note the restriction to low energies for the emitted quantum. In fact we assume that $\omega \to 0$ is the dominant limit, such that $\omega e^{\alpha_i} \to 0$, even though we must take α_i large to approach the extremal hole. Thus we do not obtain the greybody factors in this calculation. The reason for this requirement on ω is the following. At step (2) in the above sequence of calculations, we had a 4-brane that carried some transverse velocity. This is a 5-brane with a transverse velocity in the 11-dimensional picture. What happens if we now boost along x^{11}, as in step 3? If the transverse velocity was zero, the 5-brane metric would remain unaffected, and the only changes in the emitted quantum would come from the change of the scale of recompactification. But if we have a transverse velocity of order ϵ, then the 5-brane that results after the boost is ''tilted'' in the $x-x^{11}$ plane, by an angle $\sim\epsilon$ from the x^{11} axis. This is the picture in the covering space, but now we are unable to do a Kaluza-Klein reduction along the x^{11} direction, since we have no translation invariance in the x^{11} direction. Thus we cannot do the recompactification. In the limit $\epsilon \to 0$ we can ignore this ''tilt,'' and do the reduction, which is what we have implicitly done. To check the above estimates, we first take the metric produced by a 5-brane with a small transverse velocity—this can be obtained from the Aichelburg-Sexl metric for a graviton, followed by T dualities. Then we boost in the x^{11} direction, and read off the ''tilt'' from the resulting solution.

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